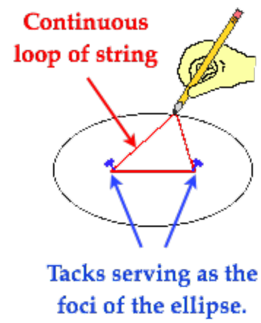
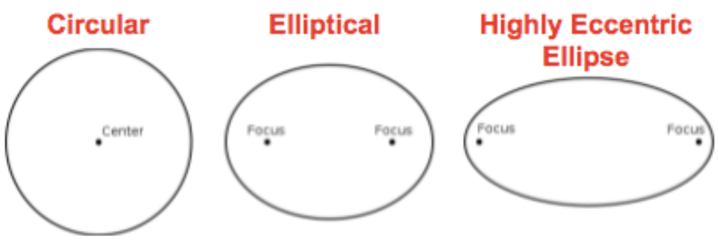
Kepler's Three Laws of Planetary Motion
Lesson Notes

Learning Outcomes
- What are Kepler's three laws of planetary motion and what is their significance?

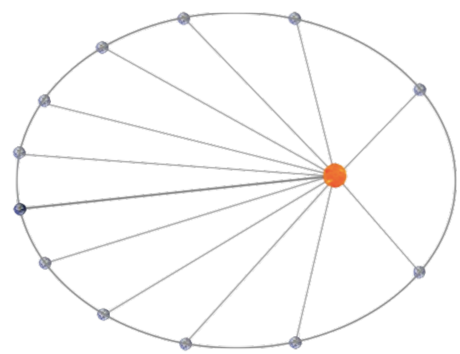
Kepler's First Law
- **The Law of Ellipses**: The path of the planets about the sun is elliptical in shape, with the center of the sun being located at one focus.
- An **ellipse** is a special curve in which the sum of the distances from every point on the curve to two other points (known as foci) is a constant.





Kepler's Second Law
The Law of Equal Areas: An imaginary line drawn from the center of the sun to the center of the planet will sweep out equal areas in equal intervals of time.

Corollary: Planets move fastest along their orbital path when they are closest to the Sun.



Kepler's Third Law
The Law of Harmonies: The ratio of the squares of the periods of any two planets is equal to the ratio of the cubes of their average distances from the sun.

$$\frac{T_{Earth}^2}{T_{Mars}^2} = \frac{R_{Earth}^3}{R_{Mars}^3} \Rightarrow \frac{T_{Earth}^2}{R_{Earth}^3} = \frac{T_{Mars}^2}{R_{Mars}^3}$$

Planet	T = Period (s)	R = Ave. Radius (m)	T^2/R^3 (s^2/m^3)
Earth	3.156×10^7	1.4957×10^{11}	2.98×10^{-19}
Mars	5.93×10^7	2.278×10^{11}	2.98×10^{-19}

Planetary Data
The third law is the only law that compares one planet's orbital characteristics to another planet's orbital characteristics.

Planet	T = Period (yr)	R= Ave. Distance (au)	T^2/R^3 (yr^2/au^3)
Mercury	0.241	0.39	0.98
Venus	0.615	0.72	1.01
Earth	1.00	1.00	1.00
Mars	1.88	1.52	1.01
Jupiter	11.8	5.20	0.99
Saturn	29.5	9.54	1.00
Uranus	84.0	19.18	1.00
Neptune	165	30.06	1.00
Pluto*	248	39.44	1.00

* Forgive us.

NOTE: The average distance value is given in **astronomical units** where 1 a.u. is equal to the distance from the earth to the sun - 1.4957×10^{11} m.

The orbital period is given in units of **earth-years** where 1 earth year is the time required for the earth to orbit the sun - 3.156×10^{7} seconds.

Newton and Kepler's Third Law

Newton reasoned that the net force acting on orbiting planets was the force of gravity. And so the following two equations are true. (**R** = ave. radius of orbit)

$$F_{net} = (M_{planet} \cdot v^2) / R \quad \Leftarrow \text{Equal} \Rightarrow \quad F_{grav} = (G \cdot M_{planet} \cdot M_{Sun}) / R^2$$

The speed of the planets is given by **2πR/T** where **T** is the period. And so …

$$(M_{planet} \cdot 4 \cdot \pi^2 \cdot R^2) / (R \cdot T^2) = (G \cdot M_{planet} \cdot M_{Sun})/R^2$$

We can simplify this equation to …

$$T^2 / R^3 = (4 \cdot \pi^2) / (G \cdot M_{Sun})$$

The right side of the equation is the same for every planet.

The Universal Nature of $T^2/R^3 = k$

Newton's derivation of the equation is not restricted to planets and the solar system. It would apply universally to any object orbiting a central body.

$$T^2 / R^3 = (4 \cdot \pi^2) / (G \cdot M_{Central})$$

Every moon orbiting Jupiter should demonstrate this:

Moon	T (days)	Ave. R (km)	T^2/R^3 (d^2/km^3)
Io	1.7691	421 700	4.17×10^{-17}
Europe	3.5512	671 034	4.17×10^{-17}
Ganymede	7.1546	1 070 412	4.17×10^{-17}
Callisto	16.689	1 882 709	4.17×10^{-17}

Show your solution to the Example Problem on Slide 10.

Example Problem - Proportional Thinking

Two planets - Planet A and Planet B - are orbiting a star. Planet A has five times the orbital radius as Planet B. How many times larger is the period of Planet A compared to the period of Planet B?