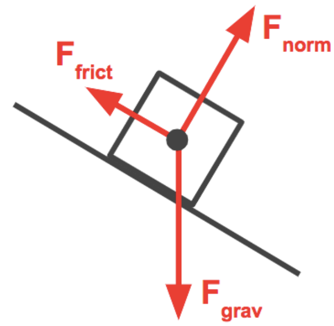
Inclined Planes
Lesson Notes

Learning Outcomes
- How do you determine the free-body diagram for an object on an inclined plane?
- How do you mathematically analyze the forces for an object on an inclined plane?

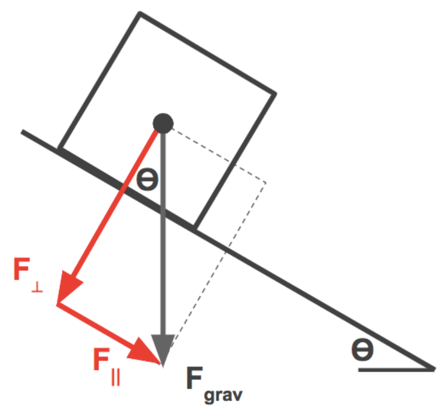
A Big Idea
How an object moves along an inclined plane depends upon the relative magnitude and direction of the forces that act upon it. Here's some forces to expect:
- Gravity (F_{grav}); always present; always downward
- Normal (F_{norm}); always present; always ⊥ to plane
- Friction (F_{frict}); usually present; always || to plane and opposite motion
- Applied (F_{app}) or Tension (F_{tens}); sometimes present



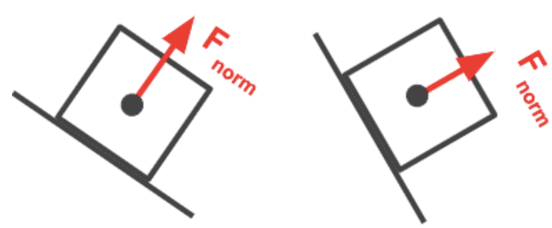
Components of Gravity
An object on an incline accelerates parallel to and down the incline. This acceleration is caused by the force of gravity. One needs to consider the components of gravity that are parallel and perpendicular to the plane.

$$F_{||} = m \cdot g \cdot \sin\Theta$$

$$F_{\perp} = m \cdot g \cdot \cos\Theta$$



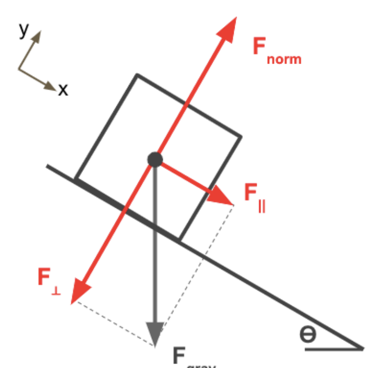
The Normal Force
The normal force (**F**) is the force of support on the object that results from the interaction between the object and the inclined plane. The normal force is the one force perpendicular to the plane that balances the perpendicular component of gravity. Most commonly:



$$F_{norm} = F_{\perp} = m \cdot g \cdot \cos\Theta$$

Simplest Case: No Friction

⊥ to the Plane	|| to the Plane	$a = F_{net} / m$
Balanced forces	Unbalanced forces	$a = \cancel{m} \cdot g \cdot \sin\Theta / \cancel{m}$
$F_{norm} = F_{\perp}$	$F_{net} = F_{\|\|} = m \cdot g \cdot \sin\Theta$	$a = g \cdot \sin\Theta$



Factoring in Friction

When an object slides down the incline, friction is directed up the incline and **$F_{frict} = \mu \cdot F_{norm}$**.

⊥ to the Plane

Balanced forces

$F_{norm} = F_{\perp} = m \cdot g \cdot \cos\Theta$

|| to the Plane

Unbalanced forces

$F_{net} = F_{||} - F_{frict}$

$F_{net} = m \cdot g \cdot \sin\Theta - \mu \cdot m \cdot g\cos\Theta$

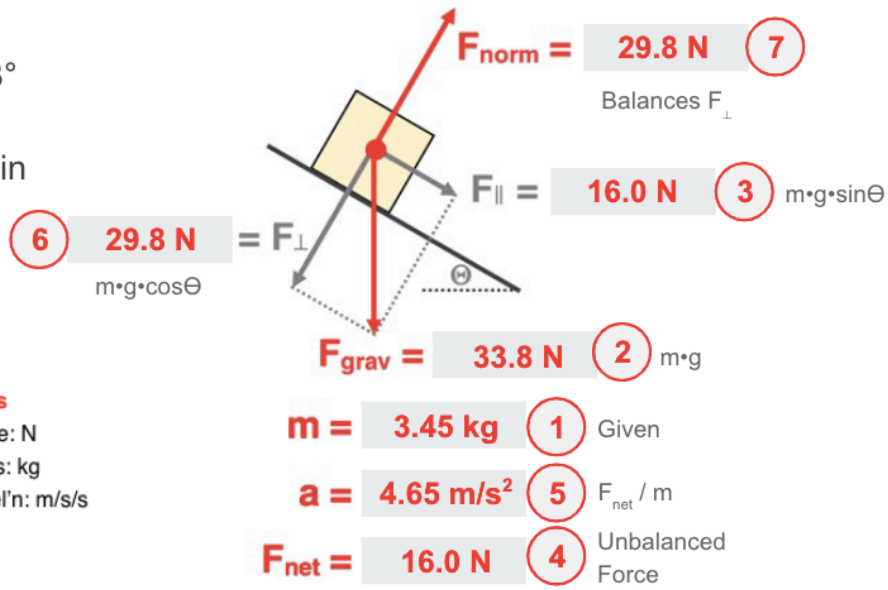
$a = F_{net} / m$

$a = (\cancel{m} \cdot g \cdot \sin\Theta - \mu \cdot \cancel{m} \cdot g\cos\Theta) / \cancel{m}$

$\mathbf{a = g \cdot \sin\Theta - \mu \cdot g \cdot \cos\Theta}$

Example 1: No Friction

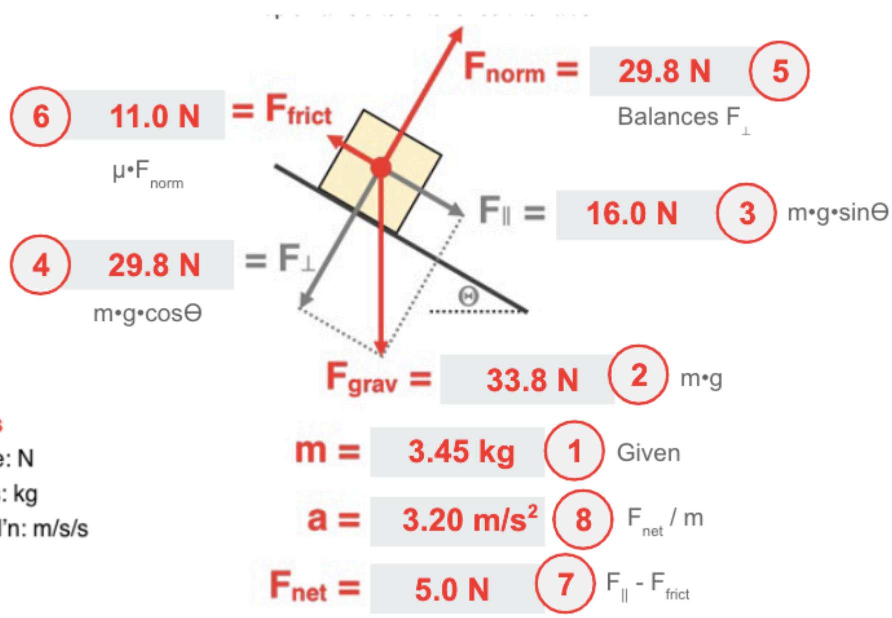
A 3.45-kg object slides down a 28.3° inclined plane. Ignore friction. Fill in all blanks and determine the acceleration.



Units
Force: N
Mass: kg
Accel'n: m/s/s

Example 2: Consider Friction

A 3.45-kg object slides down a 28.3° inclined plane with a μ value of 0.371. Fill in all blanks and determine the acceleration.



Units
Force: N
Mass: kg
Accel'n: m/s/s