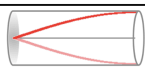
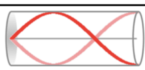
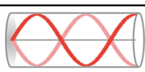
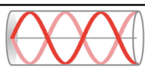
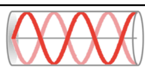
Mathematics of Closed-End Air Columns
Lesson Notes

Learning Outcomes
- What are the formulas one needs to solve a problem involving an closed-end air column?
- How does one solve an closed -end air column problem?

Closed-End Air Columns – Math Relationships
Know: wave patterns, relationships, and formulas
Have a strategy!

Harmonic	Pattern	# of Nodes	# of Antinodes	λ	f	Examples λ (m)	Examples f (Hz)
1st		1	1	λ_1	f_1	2.40	150
3rd		2	2	$\lambda_1/3$	$3 \cdot f_1$	0.80	300
5th		3	3	$\lambda_1/5$	$5 \cdot f_1$	0.48	450
7th		4	4	$\lambda_1/7$	$7 \cdot f_1$	0.343	600
9th		5	5	$\lambda_1/9$	$9 \cdot f_1$	0.267	750
nth	--	(n+1)/2	(n+1)/2	λ_1/n	$n \cdot f_1$	2.40/n	150•n

For a closed-end air column of length **L**:

$$\lambda_n = \lambda_1/n$$

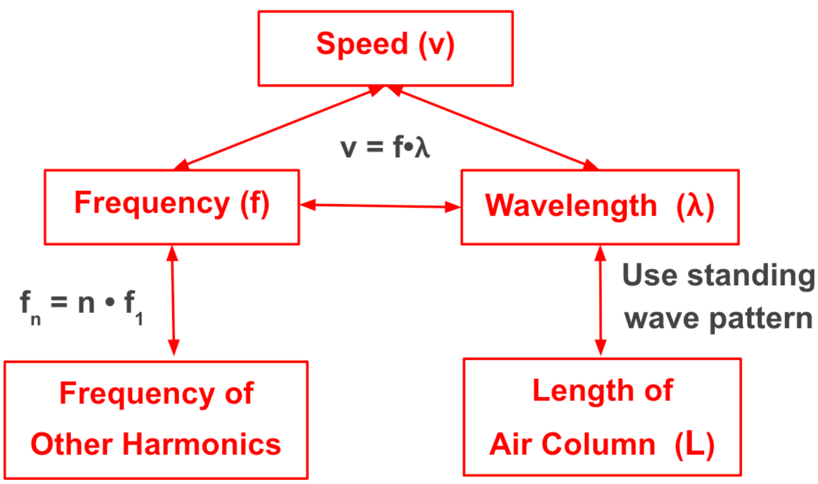
$$f_n = n \cdot f_1$$

$$v = f \cdot \lambda$$

$$\lambda = (4/n) \cdot L$$

n = harmonic #

An Effective Strategy

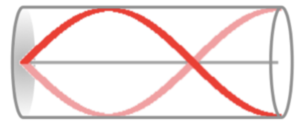


Follow through the video and provide worked-out solutions to the following problems:

Example 1: Solving for f_n from f_1
A closed-end air column has a 1st harmonic of 125 Hz. What is the frequency of the next three harmonics?

Example 2: Solving for f_1 from f_n

A closed-end column has a frequency of 360 Hz and vibrating as shown. What is the frequency of the 1st harmonic?



Example 3: Solving for λ or L from v and f

A closed-end air column resonates with its seventh harmonic frequency of 882 Hz. The speed sound is 344 m/s. Determine the length of the air column.

Example 4: Solving for f_1 or f_n from v and L

Determine the first three harmonic frequencies of a 62.0-cm closed-end air column. The speed of sound is 340 m/s.

Example 5: Solving for v from f and L

A 72-cm long closed-end air column resonates with its fifth harmonic at a frequency of 590 Hz. Determine the speed of sound in the air column.

Other Variations

Depending on your course (level, organization of topics, etc.), open-end air column problems can have several variations from the examples.

For sound waves in air: **$v = 331\ \text{m/s} + (0.6\text{m/s/°})\cdot T$** (**T** = temperature in °C)