

Mathematics of Open-End Air Columns Lesson Notes

Learning Outcomes

- What are the formulas one needs to solve a problem involving an open-end air column?
- How does one solve an open-end air column problem?

Open-End Air Columns – Math Relationships

Know: wave patterns, relationships, and formulas

Have a strategy!

For an open-end air column of length **L**:

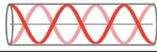
$$\lambda_n = \lambda_1/n$$

$$f_n = n \cdot f_1$$

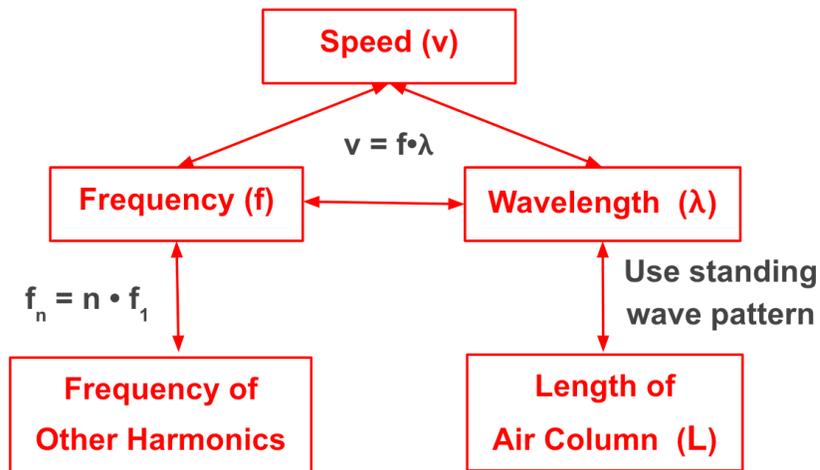
$$v = f \cdot \lambda$$

$$\lambda = (2/n) \cdot L$$

n = harmonic #

Harmonic	Pattern	# of Nodes	# of Antinodes	λ	f	Examples	
						λ (m)	f (Hz)
1 st		1	2	λ_1	f_1	1.20	280
2 nd		2	3	$\lambda_1/2$	$2 \cdot f_1$	0.60	560
3 rd		3	4	$\lambda_1/3$	$3 \cdot f_1$	0.40	840
4 th		4	5	$\lambda_1/4$	$4 \cdot f_1$	0.30	1120
5 th		5	6	$\lambda_1/5$	$5 \cdot f_1$	0.24	1400
n th	--	n	n+1	λ_1/n	$n \cdot f_1$	1.20/n	280·n

An Effective Strategy



Follow through the video and provide worked-out solutions to the following problems:

Example 1: Solving for f_n from f_1

An open-end air column has a 1st harmonic of 128 Hz. What is the frequency of the next three harmonics?

Example 2: Solving for f_1 from f_n

An open-end column has a frequency of 360 Hz and vibrating as shown. What is the frequency of the 1st harmonic?



Example 3: Solving for λ or L from v and f

An open-end air column resonates with its fourth harmonic frequency of 488 Hz. The speed of sound is 345 m/s. Determine the length of the air column.

Example 4: Solving for f_1 or f_n from v and L

Determine the first three harmonic frequencies of a 1.05-m open-end air column. The speed of sound is 342 m/s.

Example 5: Solving for v from f and L

A 92.0-cm long open-end air column resonates with its fifth harmonic at a frequency of 921 Hz. Determine the speed of sound in the air column.

Other Variations

Depending on your course (level, organization of topics, etc.), open-end air column problems can have several variations from the examples.

For sound waves in air: $v = 331 \text{ m/s} + (0.6\text{m/s}^\circ)\cdot T$ (T = temperature in $^\circ\text{C}$)