Adding Right Angle Vectors

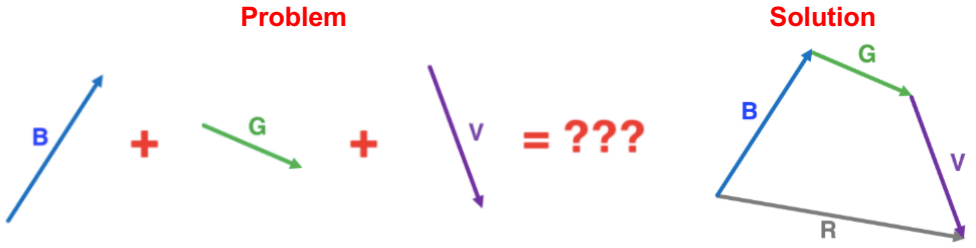
Lesson Notes

What is VectorAddition?

The process of combining two or more vectors to determine the cumulative effect.

Head-to-Tail Method

When adding vectors, place the tail of the second vector at the head of the first vector. The tail of the third vector is placed at the head of the second vector. The **resultant** vector is drawn from the tail of the first vector to the head of the last vector.



Adding Two Right Angle Vectors

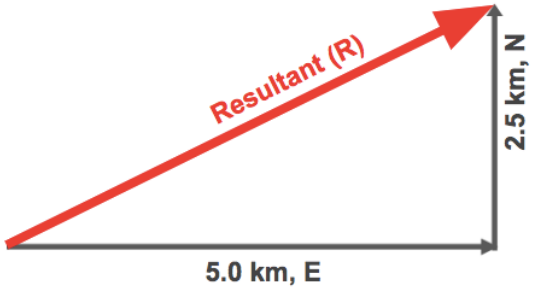
When two right angle vectors are added, the resultant is the hypotenuse of a right triangle.

Add **A** + **B** where

A: 5.0 km, East

B: 2.5 km, North

The magnitude of the resultant (**R**) is calculated using the **Pythagorean theorem**.



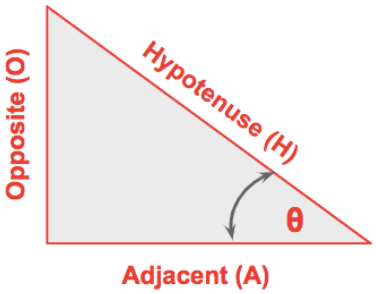
$R^2 = (5.0\ km)^2 + (2.5\ km)^2 = 31.25\ km^2$

$R = \sqrt{(31.25\ km^2)}$

R = 5.6 km . . (5.5901… km)

Trigonometric Functions

Sine, **cosine**, and **tangent** are trigonometric functions that relate an angle of a **right triangle** to the lengths of its sides. Trigonometric functions can be used to determine the direction of the resultant (R).



SOH CAH TOA

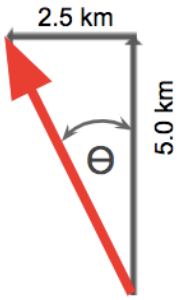
Sin θ = Opposite/Hypotenuse

Cos θ = Adjacent/Hypotenuse

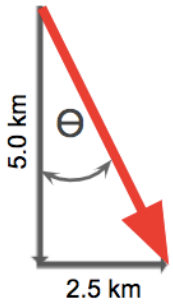
Tan θ = Opposite/Adjacent

The Angle is NOT the Direction

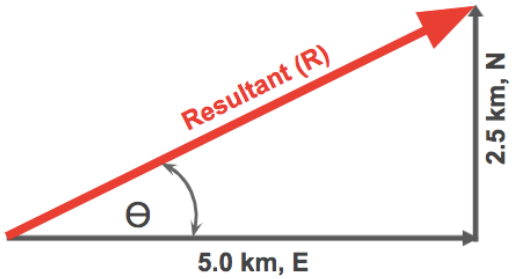
The angle is used to find the direction; but the angle isn't necessarily the direction. This is particularly the case for 2nd, 3rd, and 4th quadrant vectors, as shown below.



Θ = 26°
Dir'n = 116°
(CCW from E)



Θ = 26°
Dir'n = 296°
(CCW from E)



tan(Θ) = opposite/adjacent

tan(Θ) = 2.5 km/5.0 km = 0.50

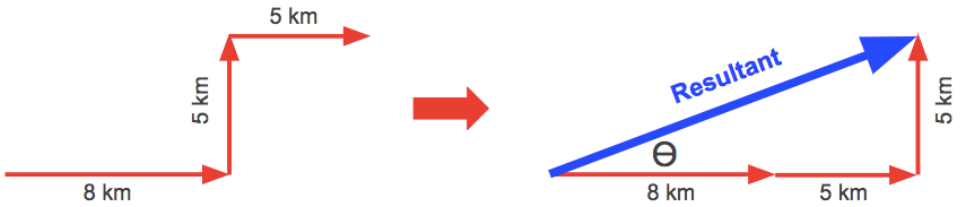
Θ = $\tan^{-1}$ (0.50) = 26.56505… °

Θ = 26° **Direction = 26° CCW**

Adding Three or More Vectors - Example

Add **A + B + C**

(**A**: 8 km, E, **B**: 5 km, N, **C**: 5 km, E)



$R^2 = (13\ km)^2 + (5\ km)^2 = 194\ km^2$

$R = \sqrt{(194\ km^2)}$

R = 14 km . . (13.928… km)

tan(Θ) = 5 km/13 km = 0.38461...

Θ = $\tan^{-1}$ (0.38461) = 21.0375… °

Θ = 21° **Direction = 21° CCW**

Summing Horizontals and Verticals

When there are more than two right angle vectors, consolidate them into two vectors. Then add the two consolidated N-S and E-W vectors to determine the overall resultant.

4 km, East
8 km, South
5 km, North
6 km, West
12 km, East

∑E-W: 10 km, East
(+4 km - 6 km + 12 km)

∑N-S: 3 km, South
(+5 km - 8 km)

∑E-W
Sum of the east-west vectors

∑N-S
Sum of the north-south vectors