The Sound of Music

The sound of many musical instruments is the result of vibrations resonating within a column of air. Air column resonance is a topic of study in the field of physics. There are two basic types of air columns – referred to as **closed-end air columns** and **open-end air columns**. The difference depends on whether one or both ends of the column are open to the surrounding atmosphere.

Any column of air has a set of frequencies at which the air particles will naturally vibrate. These frequencies are called **harmonics**; their value is dependent upon the length of the air column. The lowest frequency in the set is known as the **first harmonic**. Other frequencies in the set are whole number multiples of the lowest frequency. The mathematical relationship between the wavelength, frequency, and speed for the various harmonics is very predictable. **Table 1** illustrates these relationships for a 60-cm long closed-end air column. **Table 2** illustrates these same relationships for a 60-cm long open-end air column.

Table 1: Closed-End Air Columns

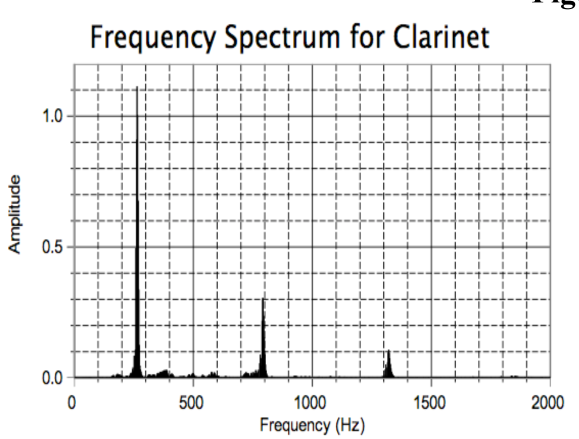
Harmonic	Frequency	Wavelength	Speed
1^{st}	142 Hz	2.40 m	340 m/s
3^{rd}	425 Hz	0.80 m	340 m/s
5^{th}	708 Hz	0.48 m	340 m/s
7^{th}	992 Hz	0.34 m	340 m/s

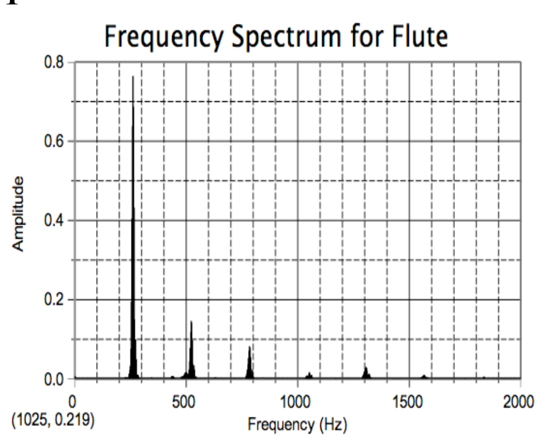
Table 2: Open-End Air Columns

Harmonic	Frequency	Wavelength	Speed
1^{st}	283 Hz	1.20 m	340 m/s
2^{nd}	567 Hz	0.60 m	340 m/s
3^{rd}	850 Hz	0.40 m	340 m/s
4^{th}	1133 Hz	0.30 m	340 m/s
5^{th}	1417 Hz	0.24 m	340 m/s

When a specific note is played on a musical instrument, the air particles immediately begin to vibrate with a set of many frequencies. These frequencies combine to produce the sound that we hear. Some of the frequencies within the set quickly dissipate and do not affect the overall sound. Other frequencies are sustained over time and become the prominent frequencies that affect the sound. These *enduring* frequencies are the harmonic frequencies. Two different instruments can play the same note, yet the resulting sounds can be quite different. A computer analysis of these sounds reveals that the difference has to do with the relative strength of the various harmonics that the instrument produces. The analysis results in a **frequency spectrum**. The spectrum shows the specific frequencies within the sound and their relative intensity or amplitude. Frequency spectra for a clarinet and a flute playing the note C_4 are shown in **Figure 1**.

Figure 1





Questions

1. Use the pattern in **Table 2** to predict the frequency of the 10^{th} harmonic of a 60-cm long open-end air column.
   a. Approximately 340 Hz
   b. Approximately 1000 Hz
   c. Approximately 1410 Hz
   d. Approximately 2830 Hz

2. Which changes are observed of the frequency, wavelength and speed as the harmonic number of any given air column is increased?
   a. The frequency, wavelength, and speed increase.
   b. The frequency and wavelength increase but the speed remains constant.
   c. The frequency and speed increase but the wavelength remains constant.
   d. The frequency increases, the wavelength decreases, and the speed is not changed.

3. Which statement accurately compares the frequencies of first harmonic for both types of air columns? Assume each air column has the same length.
   a. Their frequencies are identical.
   b. The frequency of the open-end air column is two times higher.
   c. The frequency of the closed-end air column is two times higher.
   d. The frequencies vary and a pattern can not be predicted.

4. The *spikes* shown in the frequency spectrum in **Figure 1** represent the harmonic frequencies of the instrument. What are the frequencies of the three strongest harmonics for the clarinet?
   a. Approximately 250 Hz, 750 Hz, and 1250 Hz.
   b. Approximately 260 Hz, 520 Hz, and 790 Hz.
   c. Approximately 260 Hz, 780 Hz, and 1320 Hz.
   d. Approximately 1100 Hz, 300 Hz, and 100 Hz.

5. Inspect the ratio of frequencies for the *spikes* of the clarinet's frequency spectrum in **Figure 1**. Compare it to the ratio of frequencies of the various harmonics shown in **Table 1** and **Table 2**. What is the evidence that exists that the clarinet consists of either a closed-end or an open-end air column?
   a. The clarinet consists of an open-end air column because all the harmonics are multiples of the first harmonic.
   b. The clarinet consists of an open-end air column because the height of the spikes in **Figure 1** is consistent with the pattern of the wavelengths in **Table 2**.
   c. The clarinet consists of a closed-end air column because the height of the spikes in **Figure 1** is consistent with the pattern of the wavelengths in **Table 1**.
   d. The clarinet consists of a closed-end air column because the frequencies of the second and third spikes are three times and five times the frequency of the first spike.

Answers and Explanations

1. **Answer:** D

   Explanation: The answer to this question requires that one extrapolate. *Extrapolation* involves estimating the value of a variable that lies outside of a stated or known range. **Table 2** lists frequency values for the first five harmonics of an open-end air column. The tenth harmonic lies outside this range and so its value must be estimated by observing the pattern in the stated range. The pattern that exists is that the frequency of each harmonic is some multiple of the first harmonic. For instance, the second harmonic frequency is two times the first harmonic frequency. And the third harmonic frequency is three times the first harmonic frequency. And finally, the fourth harmonic frequency is four times the first harmonic frequency. Extending this pattern to the tenth harmonic frequency leads to the conclusion that is must be 10 times greater than the first harmonic frequency of 283 Hz. And so choice D (2830 Hz) is the correct answer.

2. **Answer:** D

   Explanation: In both **Table 1** and **Table 2**, the same pattern is observed. As the harmonic number increases (going down the first column), the frequency increases (going down the second column), the wavelength decreases (going down the third column) and the speed (fourth column) is remains the same value. This makes choice D the only answer.

3. **Answer:** B

   Explanation: Both **Table 1** and **Table 2** must be used to answer this question. The first harmonic of the closed-end air column (first row of **Table 1**) must be compared to the first harmonic of the open-end air column (first row of **Table 2**). One observes that the open-end air column has a frequency (283 Hz) that is approximately two times the frequency of the closed-end air column (142 Hz). Choice B is the best answer.

4. **Answer:** C

   Explanation: **Figure 1** shows the frequency spectrum with the prominent frequencies shown as spikes on the graph. The clarinet is shown in the graph on the left. The *most prominent peaks* are the ones that are highest (or strongest). Reading off the horizontal axis, one observes that these three peaks occur at approximately 250 Hz (or a little greater), approximately 800 Hz (or a little less) and at approximately 1300 Hz (or a little more). These readings are consistent with choice C.

5. **Answer:** D

   Explanation: The data in **Table 1** show that the lowest three harmonics have frequency ratios of 1:3:5. This feature distinguishes closed-end air columns from the open-end air columns shown in **Table 2** where the frequency ratios are 1:2:3. The frequency spectrum of the clarinet shows in **Figure 1** shows ratios that are consistent with the closed-end air columns. The three peaks of the spectrum have frequencies of approximately 260 Hz, 780 Hz, and 1300 Hz. These peaks have a 1:3:5 ratio of frequencies. This makes choice D the best answer.