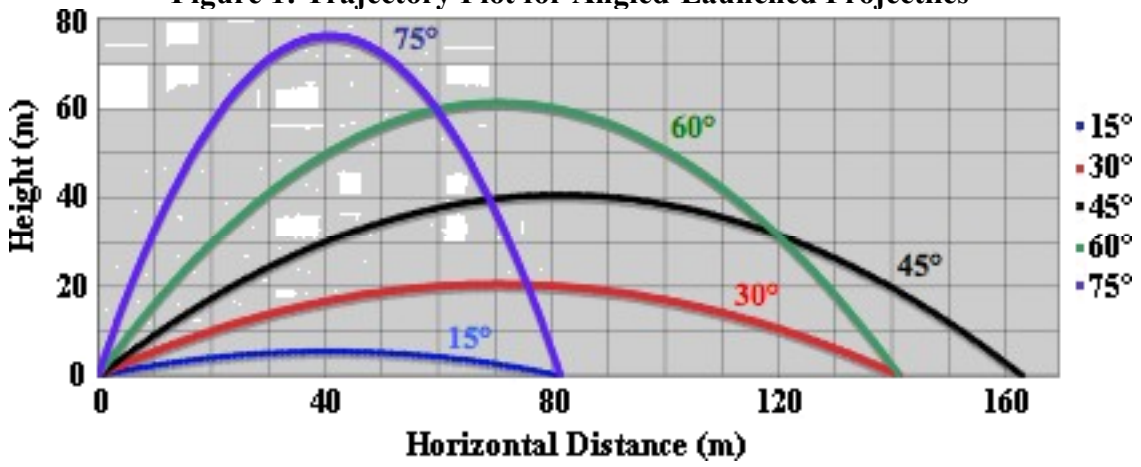
Maximum Range Explained

A projectile is an airborne object that is under the sole influence of gravity. As it rises and falls, air resistance has a negligible effect. The distance traveled horizontally from the launch position to the landing position is known as the **range**. The range of an angled-launch projectile depends upon the launch speed and the launch angle (angle between the launch direction and the horizontal). **Figure 1** illustrates the effect of launch angle on the range of a projectile with a launch speed of 40.0 m/s and five different launch angles.

Figure 1: Trajectory Plot for Angled-Launched Projectiles



A projectile is often thought of as having two independent and simultaneous motions - a motion in the horizontal direction and a motion in the vertical direction. Gravity, being a vertical force, affects only the vertical motion. When launched, a projectile acquires a distinctly different horizontal and vertical speed; the actual values depend on the launch angle. During its flight through the air, the horizontal speed of the projectile remains constant; the vertical speed changes due to gravity. The horizontal distance depends on two factors: the horizontal speed (v_{ox}) and the time the projectile has been in the air. The horizontal distance is the product of these two quantities. The height of the projectile depends on the original vertical speed (v_{oy}) and the time that the projectile has been in the air. **Table 1** summarizes several mathematical quantities for a projectile launched with an original speed of 40.0 m/s.

Table 1: Mathematical Parameters for a 40.0 m/s Launch Speed at Various Angles

Angle (°)	v_{ox} (m/s)	v_{oy} (m/s)	t_{peak} (s)	Max. Height (m)	t_{total} (s)	Range (m)
15	38.6	10.4	1.06	5.5	2.11	82
30	34.6	20.0	2.04	20.4	4.08	141
40	30.6	25.7	2.62	33.7	5.25	161
45	28.3	28.3	2.89	40.8	5.77	163
50	25.7	30.6	3.13	47.9	6.25	161
60	20.0	34.6	3.53	61.2	7.07	141
75	10.4	38.6	3.94	76.2	7.89	82

Note: t_{peak} refers to the time it takes the projectile to reach its peak (i.e., highest point).
t_{total} refers to the total time it takes the projectile to travel through the air.
The **Max. Height** is the height that the projectile has when it is at its highest point.

Questions:

1. Using a launch speed of 40.0 m/s and any angle between 0 and 90 degrees, what would be the largest possible range for a projectile?
   a. 45 meters
   b. 90 meters
   c. 163 meters
   d. 180 meters

2. Based on the given launch parameters, which projectile will reach the highest peak?
   a. Launch speed = 40.0 m/s; launch angle = 30°
   b. Launch speed = 40.0 m/s; launch angle = 45°
   c. Launch speed = 40.0 m/s; launch angle = 60°
   d. Launch speed = 40.0 m/s; launch angle = 85°

3. Which one of the following rules regarding the relationship between the launch angle and the range seems to best fit the data in **Figure 1** and **Table 1**?
   a. Any two angles that have a 2:1 ratio will result in the same range when launched at the same speed.
   b. Two launch angles that add to 90° will result in the same range when launched at the same speed.
   c. Launch angles that are evenly divisible by 10 will result in a shorter range than those that are not evenly divisible by 10.
   d. If a large launch angle is evenly divisible (without a remainder) by a small launch angle, then their range will be the same.

4. A golfer is planning to club a ball towards the green but finds a large oak tree to be an imposing obstacle in his way. The trunk of the tree is 50 meters from the golfer. The canopy of the tree can be approximated as a circle with a radius of 15 meters. It extends to a height of 30 meters. Which listed launch angle will allow the golfer to direct the ball over the topmost branches of the tree and still drive the ball as far as possible? Assume a 40.0 m/s *launch* speed.
   a. 30 degrees
   b. 40 degrees
   c. 45 degrees
   d. 60 degrees

5. A student observes the following correlation: the time a projectile is in the air (t_{total}) increases as the original horizontal speed decreases (v_{ox}) and as the original vertical speed (v_{oy}) increases. The student wishes to determine which factor (v_{ox} and v_{oy}), if any, is related to t_{total} in a cause-effect manner. Which experiment could be performed to resolve the issue?
   a. Launch projectiles with random angles and speeds and measure t_{total}.
   b. Launch projectiles at various angles from the top of a cliff and measure t_{total}.
   c. Launch projectiles at one angle with various launch speeds and measure t_{total}.
   d. Launch projectiles straight up in the air at various vertical speeds and measure t_{total}.