Mirror Equation Derivation
Teacher's Guide

Topic:

Reflection and Mirrors

The following information is provided to the student:

> **Question:**
> How can geometry, algebra and simple ray construction be used with a concave mirror in order to develop a relationship between the object distance, image distance and focal length?
>
> **Purpose:**
> To use geometry, algebra and simple ray construction in order to develop a mathematical equation relating the object distance, image distance and focal length.
>
> A complete lab write-up includes a Title, a Purpose, a Data section, and a Conclusion. The Data section should include the provided graphic with the completed ray diagram. Strategic locations on the diagram should be clearly labeled with a letter. The derivation of the mirror equation should follow. Similar right triangles should be identified; algebraic manipulations should be shown in an organized, step-by-step fashion.

Materials Required:

Diagram of concave mirror with principal axis, focal point and an object arrow (see Auxiliary Materials section); ruler or straight edge.

Description of Procedure:

Students draw a ray diagram to determine the location of the image. The complete image is drawn and its two extremities are labeled with letters (for instance, I' and I). A final ray is drawn from the top of the object (O'); it reflects off the mirror (according to the law of reflection) at location A and travels through the image of the arrowhead (I'). The diagram is now used to find two sets of similar triangles. From the two sets of similar right triangles, two equations are written in which the ratio of corresponding sides are set equal to each other. The two sets of equations are algebraically manipulated in an effort to derive the mirror equation.

Alternative Materials and Procedure:

Alternative materials and procedures are not recommended.

Safety Concern:

There is always a higher than usual level of risk associated with working in a science lab. Teachers should be aware of this and take the necessary precautions to insure that the working environment is as safe as possible. Student *horseplay* and off-task behaviors should not be tolerated.

Suggestions, Precautions, Notes:

1. It is difficult to consider this activity as a lab; there are no observations, no measurements, and no collected data. It is simply a geometric proof. Nonetheless, the activity is a representation of one of

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> the means by which physicists generate knowledge - using known behaviors of nature and mathematical extensions in order to derive new relationships.

2. This activity is a challenging activity which should be reserved for your most able students. Considerable guidance should be provided.
3. The two sets of similar triangles are: ΔO'OA and ΔI'IA and ΔFO'O and ΔFBA. Point B is the point where the principal ray from O' traveling through the focal point intersects with the mirror (just below point A). From these two similar triangles, it can be stated that:
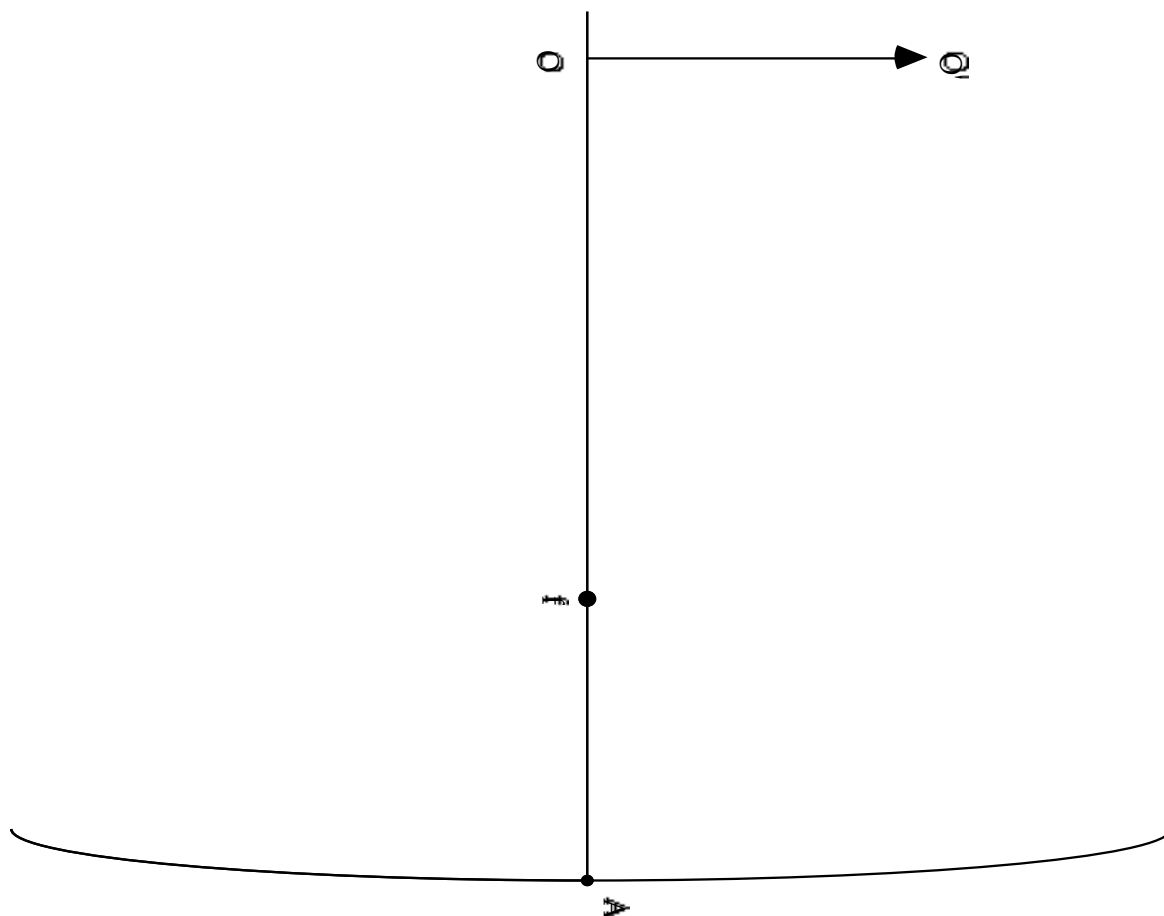
$$\frac{\overline{O'O}}{\overline{I'I}} = \frac{\overline{OA}}{\overline{IA}} \qquad\qquad \frac{\overline{O'O}}{\overline{BA}} = \frac{\overline{OF}}{\overline{FA}}$$

These line segments can be related to the object and image heights and distances and to the focal length as follows:

$$\mathbf{O'O} = h_{object}, \ \mathbf{I'I} = h_{image}, \ \mathbf{OA} = d_{object}, \ \mathbf{IA} = d_{image}, \ \mathbf{BA} = h_{image}, \ \mathbf{FA} = f, \ \mathbf{OF} = d_{object} - f$$

Auxiliary Materials:

The following page is provided to the student for completion and inclusion in the Data section of their lab notebook.

Scoring Rubric:

RM11. Mirror Equation Derivation	Score
_____ Included, labeled and organized all parts of the lab report.	

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____ Data section included provided and completed graphic; two sets of similar triangles are identified, labeled and subsequently used in a derivation. Derivation is annotated and briefly explained. Algebraic steps are shown in systematic manner leading up to the final equation relating object distance, image distance and focal length. ____ Conclusion states the equation relating the object distance, image distance and focal length.	____/____

Connections to The Physics Classroom Tutorial:

The following reading is a suitable accompaniment to this lab:

http://www.physicsclassroom.com/Class/refln/u13l3f.cfm

http://www.physicsclassroom.com/Class/refln/u13l4d.cfm

Connections to Minds on Physics Internet Modules:

Sublevels 7 and 10 of the Reflection and Mirrors module are suitable accompaniments to this lab:

http://www.physicsclassroom.com/mop/module.cfm