Musical Scales Analysis

Teacher's Guide

Topic:

Sound and Music

The following information is provided to the student:

Question:
How are the frequencies of the sounds produced by the keys on the piano keyboard mathematically related?

Purpose:
To develop a mathematical equation which relates the frequency played by the n^{th} key (white or black) on the piano keyboard relative to any given key.

A complete lab write-up includes a Title, a Purpose, a Data section, and a Conclusion/Discussion of Results. The Data section should include the provided table. Some form of numerical analysis (a data plot with linear, power or logarithmic analysis, trial and error equation testing, and/or the use of additional columns of calculated data) should be performed to determine an equation which relates the frequencies of the various keys. The numerical analysis should be clearly documented. The Conclusion/Discussion should report the equation relating the frequency to the key number. An example of how the equation can be used to calculate the frequency of the n^{th} key should be provided and explained.

Materials Required:

Frequency cheat sheet (provided below); calculator.

Description of Procedure:

Students are provided with a lengthy list of frequency values associated with the sounds played by a variety of keys on a toy piano keyboard. Students mathematically analyze the set of data in an effort to determine a pattern. Any method of analysis is allowed - inspection, finding ratios, graphical analysis, etc. Students attempt to determine a pattern and to write an equation which could be used to predict the frequency of the n^{th} key on the keyboard.

Alternative Materials and Procedure:

Alternative materials and procedures are not recommended.

Safety Concern:

There is always a higher than usual level of risk associated with working in a science lab. Teachers should be aware of this and take the necessary precautions to insure that the working environment is as safe as possible. Student *horseplay* and off-task behaviors should not be tolerated.

Suggestions, Precautions, Notes:

1. The mathematical analysis of this data is difficult, making this lab one to be used with only your most able students. If more guidance is needed, you might suggest that students find the ratio of

frequency for two adjacent keys. Repeating this for several keys results in the observation that each key is approximately 1.0595 times higher in frequency than the frequency of the previous key. An equation can be written with this information. If attempting a graphical analysis, it can be suggested that students plot the logarithm of frequency (**f**) versus key number (**K**). The following equation can be generated:

$$\log f = 0.0251 \bullet K + 2.22$$

This equation can be mathematically manipulated to take the form of

$$f = 166 * 1.0595^{K}$$

Once more, the 1.0595 multiplier is found in the equation.

2. This lab can be related to the previous lab - The Musical Intervals Lab. In that lab, pleasing sounds were those with frequency ratios of 5:4, 4:3, and 3:2 (and also 2:1, though this was not part of the lab). These frequency ratios are associated with keys which are spaced apart on the keyboard by 4, 5, 7 and 12 keys (with black keys being included in the count). Raising the 1.0595 multiplier to the powers of 4, 5, 7 and 12 produces the numbers 1.25 (5:4), 1.33 (4:3), 1.50 (3:2) and 2.00 (2:1). These are the same ratios associated with pleasing sounds. It was Pythagorus, an early Greek mathematician and natural philosopher, who suggested that "music is both pleasing to the ear and to the mathematical mind."

Auxiliary Materials:

The following page is provided to the student for completion and inclusion in the Data section of their lab notebook.

Key #	Note	Frequency (Hz)
1	F_3	174.6
2	$F_3^{\#}$	184.8
3	G_3	195.9
4	A_3^{b}	207.7
5	A_3	220.0
6	B_3^{b}	233.2
7	B_3	246.9
8	C_4	261.7
9	$C_4^{\#}$	277.5
10	D_4	293.8
11	E_4^{b}	311.0
12	E_4	329.3
13	F_4	348.7
14	F_4#	370.5
15	G_4	392.2
16	A_4^{b}	414.5
17	A_4	439.9
18	B_4^{b}	466.5
19	B_4	494.1
20	C_5	522.0

21	**$C_5^{\#}$**	**553.7**
22	**D_5**	**586.2**
23	**E_5^{b}**	**622.0**
24	**E_5**	**658.0**
25	**F_5**	**699.8**
26	**$F_5\#$**	**741.5**
27	**G_5**	**781.3**
28	**A_5^{b}**	**833.8**
29	**A_5**	**876.8**
30	**B_5^{b}**	**933.0**
31	**B_5**	**989.2**
32	**C_6**	**1050.4**

Scoring Rubric:

S9.	**Musical Scales Analysis**	**Score**
____	Included, labeled and organized all parts of the lab report.	____/____
____	Data section includes the provided table (taped in) and some form of numerical analysis in an effort to determine the mathematical equation. The numerical analysis is thoroughly and clearly documented. Is accurate and reveals good problem-solving ability.	
____	Conclusion/Discussion reports the mathematical equation; equation is accurate and clear. Thoroughly and accurately explained how the equation can be used to determine the frequency of the n^{th} key relative to the frequency of a given key.	

Connections to The Physics Classroom Tutorial:

The following reading is a suitable accompaniment to this lab:

http://www.physicsclassroom.com/Class/sound/u11l3a.cfm

Connections to Minds on Physics Internet Modules:

There are no sublevels of Minds on Physics which pertain to the topic of musical scales.